SM223 – Calculus III with Optimization — Fall 2017
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# Lesson 4. The Dot Product, cont.

## 1 Warm up

**Example 1.** Consider the triangle with vertices $P(2,0)$, $Q(0,3)$ and $R(3,4)$.

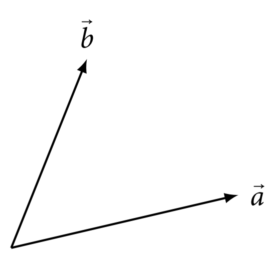
a. Find $\overrightarrow{PQ}$ (the vector starting at $P$ and ending at $Q$), $\overrightarrow{QR}$ and $\overrightarrow{PR}$.
b. Find the angle $\angle PQR$.

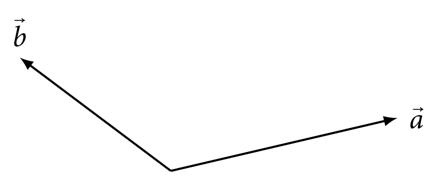
## 2 Today...

- Projections
- Practice with vectors and dot products

## 3 Projections

- **Vector projection** of $\vec{b}$ onto $\vec{a}$:




  - Denoted by $\text{proj}_{\vec{a}}\vec{b}$
  - "Shadow" of $\vec{b}$ onto $\vec{a}$

- **Scalar projection** of $\vec{b}$ onto $\vec{a}$ = signed magnitude of $\text{proj}_{\vec{a}}\vec{b}$
  - Also called the **component** of $\vec{b}$ along $\vec{a}$
  - Denoted by $\text{comp}_{\vec{a}}\vec{b}$
- The scalar and vector projections can be computed using dot products:
  - $\text{comp}_{\vec{a}}\vec{b} =$
  - $\text{proj}_{\vec{a}}\vec{b} =$

**Example 2.** Find the scalar projection and vector projection of $\vec{b} = \langle 3, 4 \rangle$ onto $\vec{a} = \langle 2, 1 \rangle$.

**Example 3.** Draw $\vec{a}$, $\vec{b}$, and $\text{proj}_{\vec{a}}\vec{b}$ from Example 2. Is the drawing what you expected?

## 4 Practice!

**Example 4.** Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2 \rangle$ onto $\vec{a} = \langle -2, 3, 1 \rangle$.

**Example 5.** Find a unit vector that is orthogonal to both $\langle 2, 0, -1 \rangle$ and $\langle 0, 1, -1 \rangle$.

**Example 6.** Determine whether the given vectors are orthogonal, parallel, or neither:

a. $\vec{a} = \langle 4, 5, -2 \rangle$, $\vec{b} = \langle 3, -1, 5 \rangle$
b. $\vec{u} = 9\vec{i} - 6\vec{j} + 3\vec{k}$, $\vec{v} = -6\vec{i} + 4\vec{j} - 2\vec{k}$